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1968 J. Phys. A: Gen. Phys. 1 55

(http://iopscience.iop.org/0022-3689/1/1/308)

J. PHYS. A (PROC. PHYS. SOC.), 1968, SER. 2, VOL. 1. PRINTED IN GREAT BRITAIN

Range fluctuations for muons of energy up to 10^6 GeV

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MS. received 7th September 1967, *in revised form* 23rd *October* 1967

Abstract. Recent measurements on cosmic rays deep underground have led to the necessity of studying the effect of fluctuations in range for muons of energy to some 10^6 GeV and ranges down to 15 000 m.w.e. The results of such calculations are given here; in these calculations, which are more extensive than hitherto, an examination is made of the effect on the muon survival probabilities deep underground of different assumptions as to the form of the cross section for the contributions from nuclear and bremsstrahlung losses and different magnitudes of the energy loss coefficients.

Survival probability curves are given from which the enhancement in underground intensity can be calculated for any sea-level muon spectrum, and results are also given of the actual enhancement factors for sea-level spectra having constant exponents.

1. Introduction

The problem of the fluctuations in range of high-energy muons in their passage through matter, generally the Earth's crust, has been studied by a number of workers, notably Bollinger (1951), Mando and Sona (1953), Zatsepin and Mikhalchi (1962), Ramana Murthy (1962), Hayman *et al.* (1963), Miyake *et al.* (1964), Nishimura (1964), Oda and Murayama (1965) and Menon and Ramana Murthy (1967). Very recently a comprehensive study has also been made by Kobayakawa (1967). The present situation is that, in the most recent works, for the same assumptions as to the form of the cross sections for the various interaction processes there is general agreement in the predicted muon survival probabilities (by survival probability is meant the probability of a muon of a particular energy at ground level 'surviving' to a particular depth underground). The agreement stems in the main from the use in these recent analyses of computers with which accurate Monte Carlo calculations can be made. Furthermore, there is now better agreement between the various authors as to the forms of the cross sections to be taken, and the result is that for most underground measurements the effect of range fluctuations can be predicted to fair accuracy, on the assumption, that is, that the muon behaves in accordance with theory.

The reason for yet another study of the range fluctuation problem is that recent measurements in the neutrino experiments in India and South Africa (Menon *et al.* 1967, Reines 1967) have demonstrated the importance of a knowledge of range fluctuations at energies and depths greater than hitherto studied. The main reason for the interest is not that the vertical depths are particularly great, but rather that importance attaches to muons of atmospheric origin arriving at significant angles to the vertical for which the slant depth of matter traversed is very great. In the present work calculations are made for depths down to 15 000 m.w.e., this corresponding to a zenith of 60° in the case of the experiment in India (vertical depth 7500 m.w.e. standard rock). The effects of adopting alternative cross sections and of varying the type of rock under consideration are also considered.

2. Adopted cross sections and method of calculation

The main processes by which muons lose energy in penetrating matter are as follows: ionization (and excitation), direct electron-pair production, bremsstrahlung and nuclear interaction. Significant fluctuations in range arise because the cross sections for the last two mentioned processes fall off rather slowly with increasing energy transfer, so that the probability of a high-energy muon losing a large fraction of its energy is not negligible. The cross sections for ionization and pair production, on the other hand, fall off so rapidly with increasing energy transfer that these processes may be considered as giving continuous energy losses.

The cross sections adopted in the present work are those given by Hayman *et al.* (1963) for 'standard' rock with $\bar{Z} = 11$, $\bar{A} = 22$, $(\overline{Z^2/A}) = 5{\cdot}5$, $\rho = 2{\cdot}65$ g cm^{-3}. In that work an approximate equation for the average rate of energy loss was given as

$$-\frac{dE}{dx} = 1{\cdot}88 + 0{\cdot}077 \ln\left(\frac{E_{\mathrm{m}}'}{mc^2}\right) + 4{\cdot}0\times10^{-6}E \quad \text{MeV g}^{-1}\text{ cm}^2$$

where E_{m}' is the maximum transferrable energy in a μ–e collision and m is the muon mass. The breakdown of the coefficient of the energy in the last term ($4{\cdot}0\times10^{-6}$ g^{-1} cm^2), which is usually denoted by b, into the fluctuating component b_{f} (bremsstrahlung and nuclear) and the non-fluctuating component b_{nf} (the remaining processes) is

$$b_{\mathrm{f}} = 2{\cdot}4\times10^{-6}, \qquad b_{\mathrm{nf}} = 1{\cdot}6\times10^{-6}.$$

It is relevant at this stage to point out that the ratio $b_{\mathrm{f}}/b_{\mathrm{nf}}$ is important in defining the magnitude of the range fluctuations, the magnitude increasing with increasing values of this ratio.

The forms of the actual cross sections used for the 'basic' calculations were as follows:

bremsstrahlung

$$\phi_1(v) = \text{const.}\,\frac{1}{v}\left(v^2 - \tfrac{4}{3}v + \tfrac{4}{3}\right) \tag{1}$$

nuclear interaction

$$\phi_2(v) = \text{const.}\,\frac{1}{v}\ln\left(\frac{1}{v}\right) \tag{2}$$

where v is the fraction of the muon energy transferred to an electron.

Those used to examine the effect of varying the cross sections will be considered later.

In the calculations depth intervals of 100 m.w.e. were taken down to 10 000 m.w.e., and of 200 m.w.e. for greater depths. A particle of a particular energy was considered to be incident vertically at sea level, and for each cell the non-fluctuating component of energy loss was subtracted together with the fluctuating component determined by a Monte Carlo technique. In calculating the latter the energy loss was replaced by a constant value if the fraction of energy lost was indicated to be less than 1/30; this procedure introduced only a negligible error, but increased the speed of calculation considerably. Calculations were made for the following sea-level energies in GeV (the number in parentheses represents the number of thousands of particles followed for that energy): 500 (1), 750 (1), 10^3 (1), $1{\cdot}5\times10^3$ (1), 2×10^3 (1), 3×10^3 (1), 5×10^3 (2), 7×10^3 (3), 10^4 (8), $1{\cdot}5\times10^4$ (3), 2×10^4 (2), 3×10^4 (2), 5×10^4 (1), 7×10^4 (1), 10^5 (1), 2×10^5 (1), 3×10^5 (1), 5×10^5 (1), 7×10^5 (1) and 10^6 (1).

3. Survival probability as a function of depth

3.1. *Results for the basic cross sections*

The survival probabilities calculated for energies in the range 10^3–10^6 GeV and depths 3000 to 15 000 m.w.e. are given in figure 1. In this figure the survival probabilities are plotted against muon energy, with depth as parameter instead of the reverse, because it is such a plot that is of value in calculating the expected intensities underground for a particular sea-level energy spectrum. Thus the underground intensity at a depth D is

$$I(D) = \int_{E_{\min}}^{\infty} P(D,E)N(E)\,dE$$

where $N(E)\,dE$ is the differential muon energy spectrum at sea level, $P(D, E)$ is the survival probability for a muon of energy E at sea level to survive to a depth D and $E_{\min}$ is the minimum energy of a muon to have any chance to survive to a depth D.

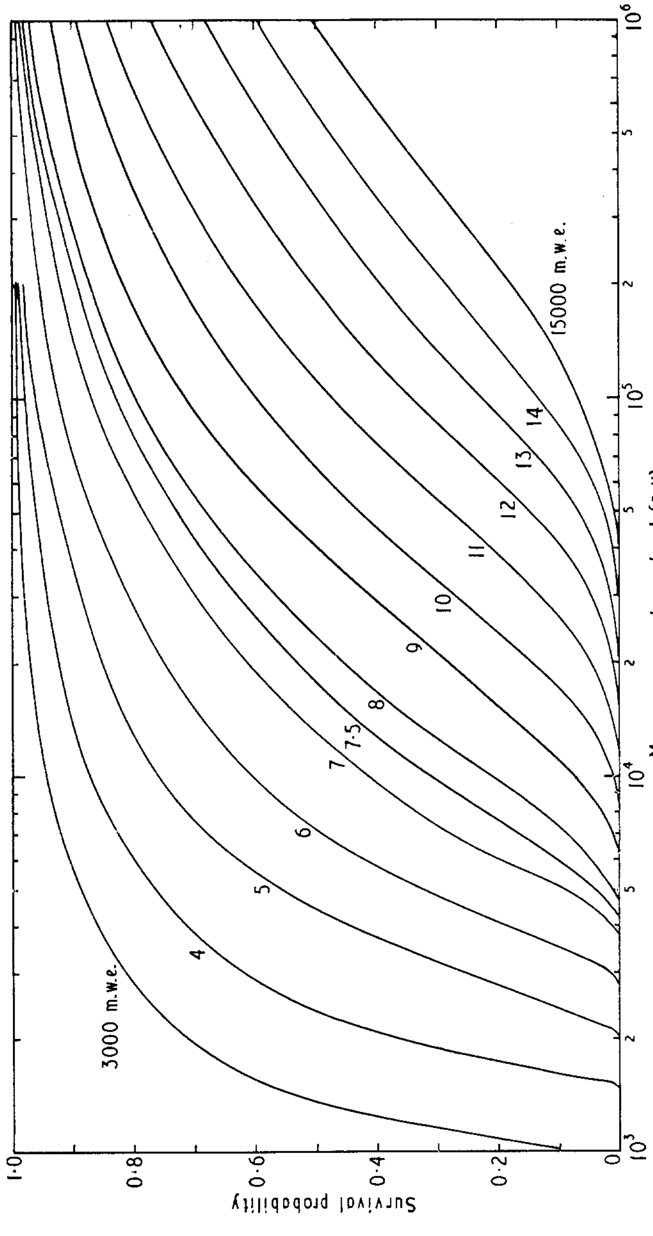

Figure 1. Survival probabilities against muon energy at sea level for a variety of depths (standard rock, $Z^2/A = 5{\cdot}5$).

In view of the non-infinite number of particles followed, there are statistical errors on the derived probabilities. At the important level of probability ($\sim 0{\cdot}2$) the fractional statistical uncertainties range from about 6% for 1000 particles to about 2·5% for 8000 particles.

3.2. *Variations of the parameters*

In a number of previous calculations, to enable analytical solutions to be found, the cross sections of § 2 ((1) and (2)) were approximated to $\phi \propto 1/v$. The corresponding survival probabilities have also been calculated in the present work, and it is found that significant differences result, particularly at survival probabilities in the region 0·5–0·9. However, it is interesting to note that, when the curves are applied to the calculation of underground intensities, the rapidly falling muon energy spectrum biases the biggest contribution to the intensity to energies where the survival probability is in the region of 10–20%. Rather fortuitously, for most depths of interest the differences between the two survival probabilities are small; for depths between 5000 and 9000 m.w.e. the absolute differences are less than about 2%.

Of greater importance are the differences consequent upon different values for the cross sections, and figure 2 shows the sensitivity of the survival probability to the overall value of b for a particular energy (10 000 GeV). A variation of b from the 'preferred' value of 4·0 can result either from a change in the composition of the rock under consideration ($b \propto \overline{Z^2/A}$) or from the adoption of different cross sections. In the important region of probability, S.P. $\simeq$ 20%, the sensitivity of the survival probability to the value of b is found to be rather large: $(\delta p/p)/(\delta b/b) \simeq 4$. In practice, this means that the determination of an accurate mean value of Z^2/A for the rock in question is important if the intensity of sea-level muons is to be derived, in the region of tens of GeV, from underground intensities.

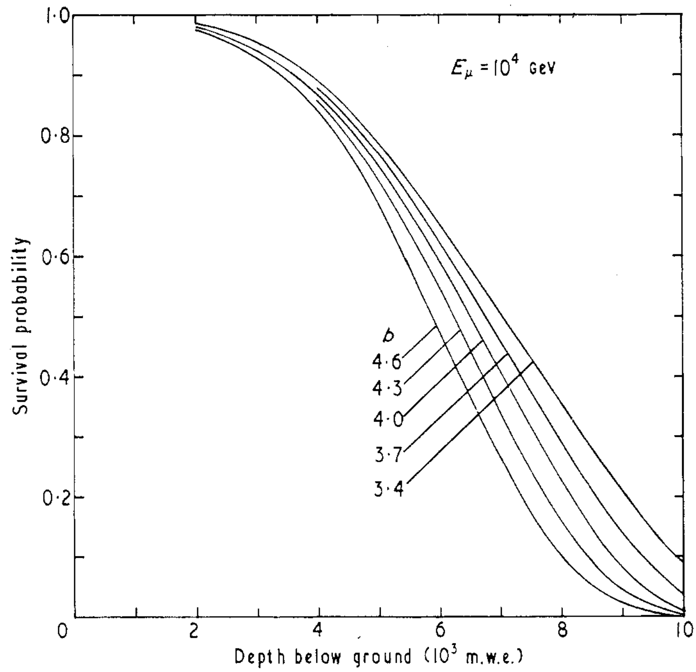

Figure 2. Sensitivity of the survival probability to the value of b for muons of 10 000 GeV.

4. Comparison with previous work

A detailed comparison of the survival probabilities calculated by the different workers has been made by Kobayakawa (1967), and it is not necessary to repeat this. Briefly, when allowance is made for the different values of b and the different ratios of b_f to b_{nf} the remaining discrepancies (in recent treatments) are barely significant.

Bearing in mind the uncertainties in the theoretical cross sections, including that arising from lack of knowledge of the behaviour of the photonuclear cross section at high energy, we conclude that the fractional error in the survival probabilities in the important region may be up to 20% for depths near 8000 m.w.e. and sea-level energies of 10^4 GeV or so. At greater energies and depths the error could be somewhat greater, the assumption throughout being that the muon behaves as expected theoretically and that no new energy loss process becomes important.

5. Fluctuation corrections to underground intensities

An 'enhancement factor' can be defined as the ratio of the number of muons to be expected at a particular depth, allowing for the effect of range fluctuations, to the number expected neglecting them, i.e.

$$F(D, \gamma) = \int_{E_{\min}}^{\infty} P(D, E) N(E)\, dE \frac{1}{N(> E_D)}$$

where E_D is the energy corresponding to the depth D on the curve of energy against mean range. The basic data from which enhancement factors may be obtained for a sea-level energy spectrum of any form are given in figure 1. As an illustration, the factors have been computed for the simplified case where the sea-level spectrum is assumed to follow a power law with constant exponent ($N(>E) = AE^{-\gamma}$) and they are given in figure 3 for a range of values of γ. The great importance of fluctuations at depths below a few thousand m.w.e. is clearly seen.

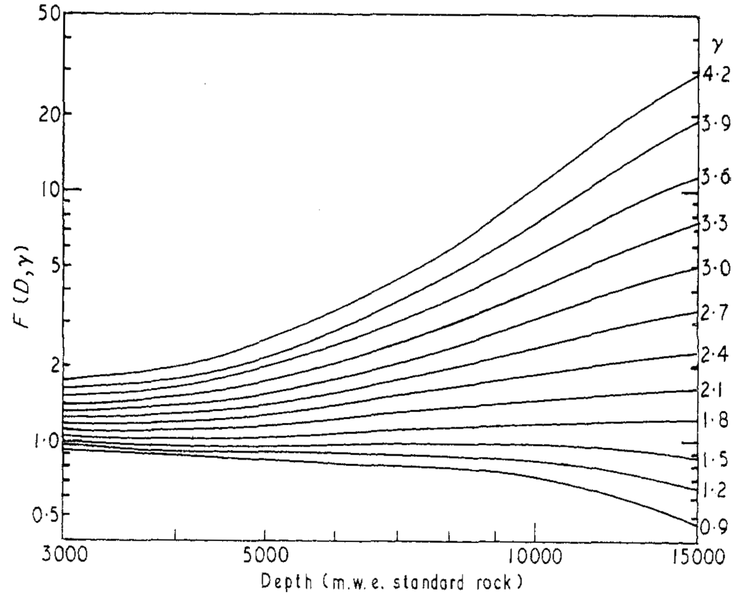

Figure 3. Enhancement factors as a function of depth, with the exponent of the integral sea-level spectrum as parameter ($Z^2/A = 5 \cdot 5$).

Acknowledgments

The authors are grateful to Dr. D. R. Creed for useful discussions. One of us (E.C.M.Y.) wishes to thank the Science Research Council for financial support.

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